A Modified Equal-Element Band-Pass Filter*

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Summary—A method is presented whereby considerable improvement in the frequency response of a five-stage, equal-element waveguide filter can be realized while preserving nearly all the structural simplicity of this realization. It is shown that by increasing the loaded Q of the center resonant element of a five-stage, equal-element filter, the pass band ripple can be appreciably reduced and the skirt selectivity improved. The modified design also provides a simple means of bandwidth adjustment.

Introduction

FABRICATION problems and cost of a waveguide band-pass filter are appreciably reduced when the structure is realized as a cascade combination of n identical sections.[1] Plots of the power transfer function show, however, that the pass band insertion loss ripple rapidly increases with n. This fact has generally limited the use of the equal-element realization to filters with less than five sections.

In the case of a five-stage filter, a considerable improvement can be achieved while still maintaining nearly all the desirable features of the equal-element realization. It will be shown that by simply increasing the loaded Q of the center resonant element, the 2-db pass band ripple inherent in the equal-element case can be appreciably reduced and the skirt selectivity improved. This may be seen intuitively by comparing the symmetrical low-pass equal-element prototype with that of an equal-ripple Tchebycheff response (see Table I).

TABLE I

Response	Prototype Ladder Elements				
	C_1	L_2	C_3	L_4	C_5
$\frac{1}{4}$ db Tchebycheff	1.414	1.318	2.241	1.318	1.414
Equal-element	1.414	1.414	1.414	1.414	1.414

It is clear that, except for the value of C_3, the five identical elements are reasonably close to the corresponding Tchebycheff values. Thus, one might reasonably expect that upon increasing the center element capacity (directly proportional to the loaded Q of the band-pass realization), the resulting structure should have a reduced pass band ripple and a skirt selectivity approaching that of the Tchebycheff unit.

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[1] G. L. Ragan, "Microwave Transmission Circuits," McGraw-Hill Book Co., Inc., New York, N. Y., pp. 681–682; 1948.

Theory

The general expression for the insertion loss of a symmetrical five-section, low-pass filter is given by

$$\text{I.L.} = 1 + w^2/4[w^4(L^2C_1^2C) - w^2(2LC_1^2 + 2LC_1C - L^2C) + (2C_1 + C - 2L)]^2 \quad (1)$$

where

w = frequency variable in radians per second
$L = L_2 = L_4$
$C_1 = C_5$, and
$C_3 = C$.

Since the only effect of multiplying the L's and C's by a constant is a change of the frequency scale, choose $L = C_1 = 1$. Substituting these values into (1), one can obtain the frequencies at which the insertion loss extrema occur:

$$w_1 = 0,$$

$$w_{2,3}^2 = \frac{1}{2}\left[\left(\frac{C+2}{C}\right) \pm \sqrt{\left(\frac{C+2}{C}\right)^2 - 4}\right], \text{ and}$$

$$w_{4,5}^2 = \frac{1}{10}\left[3\left(\frac{C+2}{C}\right) \pm \sqrt{9\left(\frac{C+2}{C}\right)^2 - 20}\right].$$

From the above equations it follows that the number of insertion loss extrema, hence pass band ripples, will depend upon the value of the center element C.

Case I.	$0 < C < 2$	5 extrema.
Case II.	$C = 2$	3 extrema (double root at $w = 1$).
Case III.	$2 < C < 4.07$	3 extrema.
Case IV.	$C = 4.07$	2 extrema.
Case V.	$C > 4.07$	1 extremum.

The theoretical response of a five-section, low-pass filter is plotted in Fig. 1 for various values of the center element C. To more clearly visualize the effect of this parameter, the curves are replotted in Fig. 2. Note the rapid reduction in the maximum pass band ripple, the broad minimum, and the uniform reduction in bandwidth as the capacity of the center element is increased.

The skirt selectivity of a modified equal-element filter has been compared to that of a Tchebycheff filter having a comparable pass band ripple. Apart from a slightly lower insertion loss for $w \gg 1$, the general behavior of the modified filter, for losses less than 20 db, follows the

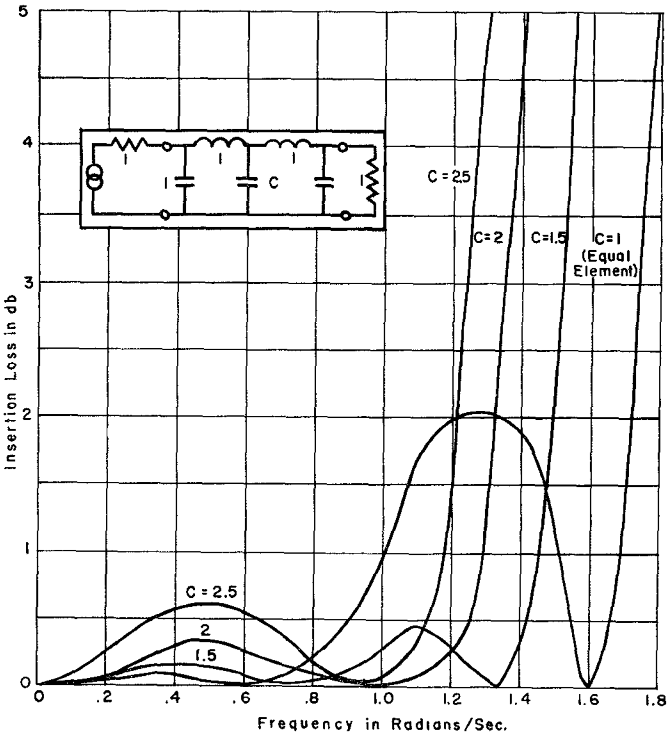

Fig. 1—Theoretical response of a five-section, low-pass filter for various values of the center element.

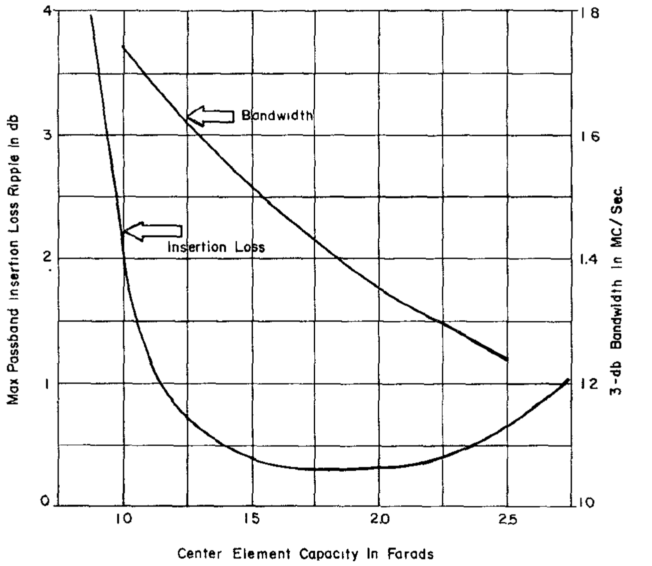

Fig. 2—Maximum pass band insertion loss ripple and a 3-db bandwidth of five-section, low-pass filter for various values of the center element.

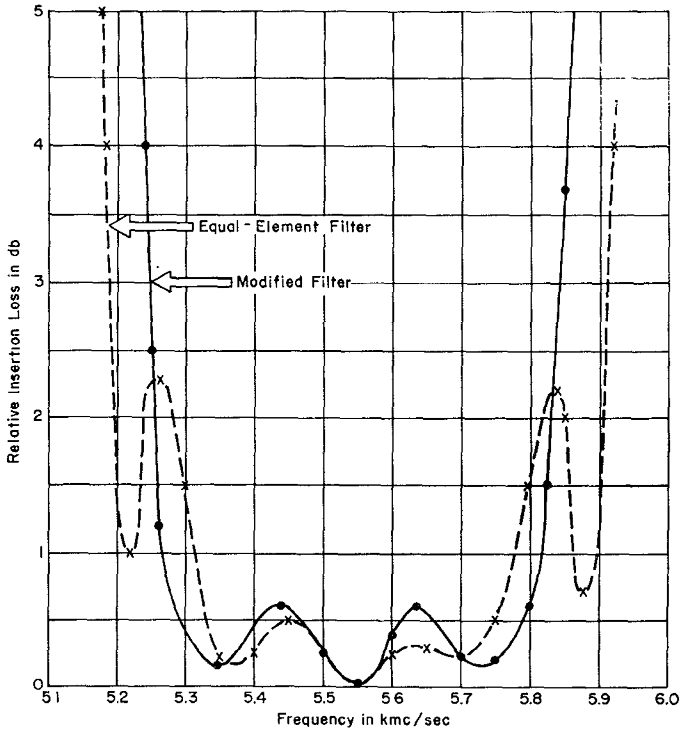

Fig. 3—Experimental response curves of a five-section, equal-element filter illustrating the effect of increasing the center resonant loaded Q.

Tchebycheff response so closely that no useful purpose is served by reproducing the curve.

A typical response of a modified five-section, equal-element band-pass filter is shown in Fig. 3. The improved response was obtained by simply replacing the inductive posts forming the center resonant element with posts of larger diameter.

Conclusion

In addition to retaining nearly all the simplicity of the equal-element realization, the modified design affords a convenient means for simply, accurately, and systematically varying the bandwidth with negligible pass band distortion. As seen in Fig. 2, the bandwidth may be varied as much as ± 10 per cent while still maintaining a ripple of less than $\frac{1}{2}$ db.

Although this technique has not been applied to higher order equal-element filters, it is felt that the general approach is applicable.